A Study of Interstitial Liquid Flow in Foam

Part III. Test of Theory

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The theory for foam drainage and overflow recently proposed by the authors' group is tested with extensive experimental data gathered from foam fractionation columns and also from stationary foams at steady state. The experimental results support the theory.

Recently the authors' group presented a theory for foam drainage and overflow (*7*). It is based on interstitial Newtonian flow through randomly oriented capillaries (Plateau borders) of noncircular cross section with surface viscosity at the capillary boundaries. It involves no empirical constants.

This theory permits the estimation of the rate at which foam flows off overhead from a foam fractionation column. Such columns are used to partially separate the constituents of solutions via differences in surface activity. Gas is bubbled up through a liquid pool at the bottom of the column, thus forming a foam which rises up the column and overflows the top. This overflow carries off solute which has been adsorbed at the bubble surfaces.

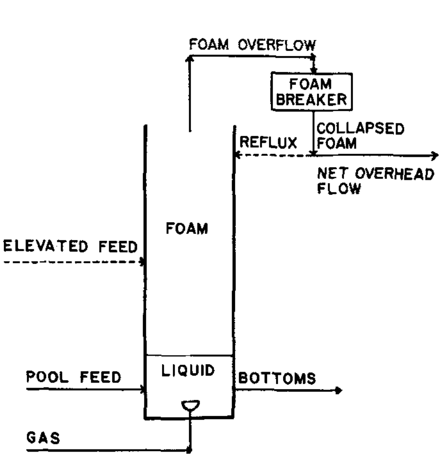

Fig. 1. Four alternative modes of flow operation with a foam fractionation column. 1. Solid lines show the simple mode. 2. The dotted reflux line is for operation as an enricher. 3. For stripping operation, pool feed is replaced with elevated feed to the foam. 4. For combined operation, reflux and elevated feed are both employed.

The degree of overhead enrichment, which may be considerable, can be increased even further through the use of internal or external reflux (*3*). The degree of stripping can be increased by injecting the feed into the foam some distance above the liquid pool (*7*). Figure 1 shows a foam fractionation column in these alternative modes of operation. The theory predicts the rate of foam overflow for all modes.

The theory incidentally also provides a relationship at steady state between the bulk density of a stationary foam in a column and the rate of deliberate liquid feed draining through the foam keeping it moist. In this situation there is no pool feed and no gas flow, and therefore no foam overflow or reflux. There is only elevated feed, stationary foam up to the feed inlet (with empty column above it), and bottoms withdrawal. This is illustrated in Figure 2.

The theory has already been compared against limited experimental data (*8*). The purpose of the current paper is to present the results of a more extensive test; details have been placed on file (*10*).

In accordance with the theory (*5, 7*) the rate of overhead flow for a fairly dry foam (meaning a foam of low liquid content) is given by Equation (1):

$$D = \frac{G^2\mu}{Ag\rho d^2}\,\phi\left(\frac{\mu^3 G}{\mu_s^2 g\rho A}\right) \qquad (1)$$

The function ϕ was evaluated theoretically in accordance with two alternatives regarding the detailed relationship between the interstitial downflow and the bulk foam upflow. The two evaluations are shown in Figure 3. The difference between them is minor, especially in the region of current interest.

Density ρ and viscosity μ are for the liquid in the capillaries. For dilute solutions these properties can be conveniently taken as those of the solvent which is usually water. Surface viscosity μ_s is a separate property which involves resistance to flow in a surface, as compared with

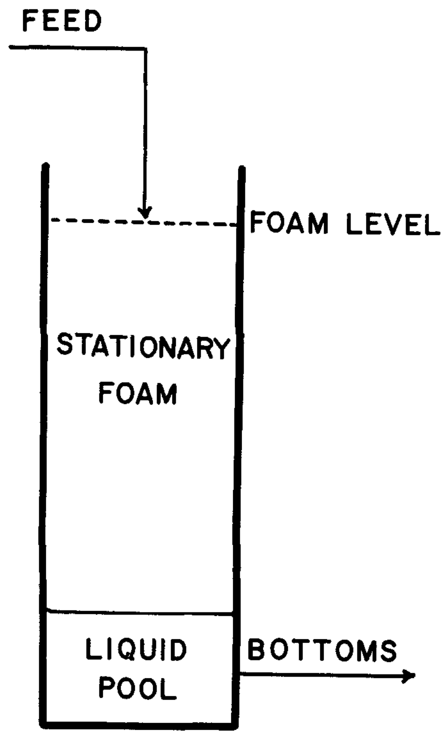

Fig. 2. Steady state drainage through a stationary foam.

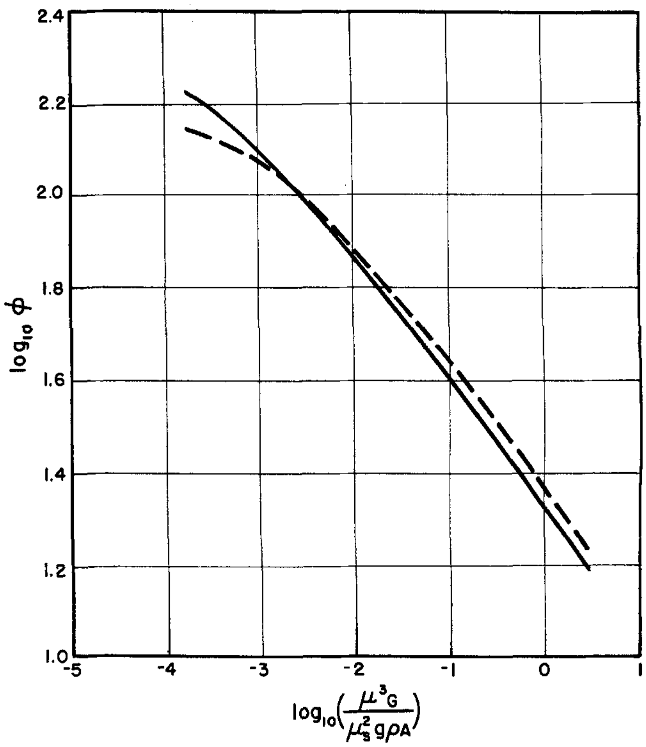

Fig. 3. Theoretical evaluation of ϕ. The solid curve is based (4, 5) on $\overline{v}_{z,\max,MO} = v_{f,SO}$. It has an asymptote of $\log_{10} \phi = 2.41$. The dashed curve is based (7) on $v_{z,\max,MO} = (3/2)\ v_{f,SO}$. It has an asymptote of $\log_{10} \phi = 2.27$.

μ which involves resistance to flow within a volume. Thus the dimensions of μ_s are force times time per length, while those for μ are of course force times time per area. The determination of μ_s is discussed later.

When there is appreciable variation in bubble diameter within the foam, an average d is employed in accordance with Equation (2):

$$d^2 = \frac{\Sigma n_i d_i^3}{\Sigma n_i d_i} \tag{2}$$

For foam of high liquid content, the theoretical procedure for estimating D is more complicated (7). However, if the foam has only a moderate liquid content, then the complicated procedure can be greatly simplified into just multiplying D from Equation (1) by the quantity $(1 + 3D/G)$ to give a corrected D. The interested reader is referred to the aforementioned earlier publications for further details.

If bubble coalescence is present within the foam column, d must be taken at the top of the column. This is important. Experiments conducted by the authors' group have shown that a small but very significant degree of coalescence can easily go unnoticed to the eye (5). It is also worth mentioning here that such coalescence furnishes internal reflux. Thus column operation which is nominally in the simple mode can in fact constitute operation in the enriching mode.

When d is not directly measured at the top of the column, it can be estimated from the average bubble diameter lower down the column by means of the ratio of corresponding bulk foam densities (7), should this ratio be known from measurement (4, 5, 8) or some other source.

For severe internal coalescence, highly nonuniform bubble sizes, or extremely high liquid content in the foam, the theoretical evaluation of ϕ breaks down since the model is based on flow through capillaries bounding regular polyhedral bubbles. For such cases, one can resort to an empirical evaluation of ϕ.

COMPARISON BETWEEN THEORY AND EXPERIMENT FOR FOAM OVERFLOW

Table 1 summarizes the data sources and results for the present test involving foam overflow. For the first three sets of data, the system employed was aqueous Triton X-100 with pool concentrations ranging from 2×10^{-7} to 20×10^{-7} g.-mole/cc. Of course with reflux the range of interstitial concentrations was even greater.

TABLE 1. APPROXIMATE RANGE OF DATA, AND RESULTS FOR THE TEST OF THEORY PRESENTED IN FIGURE 4

Set	Reference	Aqueous System	Feed inlet	External reflux ratio	d, cm.	G, cc./sec.	D, cc./sec.	A, sq. cm.	μ_s, (dyne)(sec.)/cm.	No. of runs	Average deviation, %
1	*6, 8*	Triton X-100	Pool	0	0.11 to 0.24	1.3 to 3.2	0.001 to 0.028	16.6	1.0×10^{-4}	5	16
2	*4, 5*	Triton X-100	Pool	0 to 13	0.10 to 0.22	1.7 to 9.6	0.002 to 0.305	16.6	1.0×10^{-4}	31	29
3	*6, 8*	Triton X-100	Foam	0	0.05 to 0.40	1.3 to 10.4	0.001 to 0.405	13.1 to 13.8	1.0×10^{-4}	10	25
4	*2, 3*	Aresket 300	Pool	0	0.042 to 0.046	1.5 to 1.7	0.20 to 0.38	2.8 to 9.1	1.8×10^{-4}	3	12
5	*1*	Essentially NaDBS	Pool	0	0.055 to 0.071	2.7 to 2.9	0.09 to 0.12	11.4	0.8×10^{-4}	10	13
									Total	= 59	

Triton X-100 is a commercial nonionic surfactant with the nominal formula C_8H_{17}-C_6H_4-$(OCH_2CH_2)_{9.7}$-OH.

For the first set of data, the column was operated in the simple mode. For the second set, nineteen runs were conducted with the column operating in the simple mode, and the remaining twelve runs were conducted with external reflux to operate the column deliberately in the enriching mode. For the third set, feed entered the foam rather than the pool so that operation was in the stripping mode.

Some of the early experiments (*6*) which yielded this third set of data involved very large slugs of gas in the foam and a very uncertain d at the tops of the columns. Such runs are not included here.

The fourth set of data is for aqueous Aresket 300 (commercial monobutyl biphenyl sodium monosulfonate) with pool concentrations ranging from 8×10^{-7} to 16×10^{-7} g.-mole/cc. This set of early results is very small owing to the fact that most of the data from which the set is drawn are unusable here. This is because for these data the bubble diameter was not measured at the top of the column and there was an undetermined degree of internal coalescence. The small set selected here constitutes those runs for which coalescence seemed the least, and furthermore represents D data extrapolated to zero column height (zero residence time) to further minimize the effect of coalescence.

The fifth set is from the work of another group of investigators who used aqueous sodium dodecylbenzene sulfonate in the range of 1.5×10^{-7} to 2.6×10^{-7} g.-mole/cc. in the presence of trace radioactive ions. Their photographs seem to show a wider range of bubble sizes within each bubble population than was usually the case for the authors' group of investigators who used special spargers to achieve more uniform bubble sizes within each run. Accordingly, with small bubbles fitting between larger bubbles, for a given bulk foam density their bubbles can be expected to have been more nearly spherical (and hence less polyhedral) than those of the authors' group. Their photographs seem to confirm this. There is also the question of internal coalescence in their foam. So, all in all, there is some doubt as to whether the theory is applicable to their case. Nevertheless, for the sake of completeness, their data are included in the present test.

The detailed bubble data required for applying Equation (2) to this fifth set were not published. Accordingly, d was obtained by dividing the spherical geometric factor 6 by their published values of average specific bubble surface. This does not give quite the same results as Equation (2) but is the best that can be done with the information available.

The surface viscosity for the first three sets of data, that is, for aqueous Triton X-100, has been measured independently (*8*) by the black spot method of Mysels (*9*). Its value was found to be 1.0×10^{-4} (dyne)(sec.)/cm. at 25°C.

Unfortunately, no μ_s measurements are known to the present authors for the systems employed in the last two data sets. Accordingly, for these two sets, μ_s was determined by applying the theory to the foam data. This was accomplished by combining Equation (1) with the wetness factor $(1 + 3D/G)$ and the solid curve of Figure 3, and then substituting for the quantities involved so as to solve for μ_s for each run. Results were then averaged. This yielded a μ_s of 1.8×10^{-4} (dyne)(sec.)/cm. for the aqueous Aresket 300, and 0.8×10^{-4} (dyne)(sec.)/cm. for the aqueous sodium dodecylbenzene sulfonate. Naturally, values of μ_s so determined incorporate not only any error in the data but also any limitations in the foam theory. On the other hand, they obviously improve the

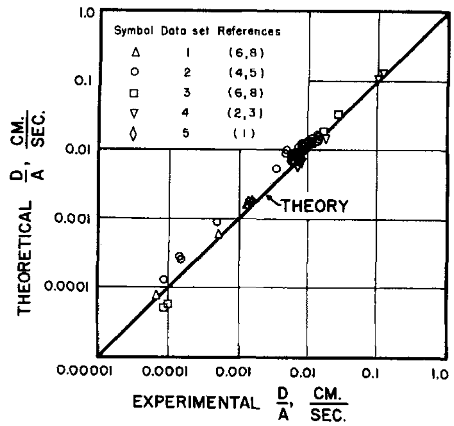

Fig. 4. Test of theory (5, 7) for predicting the foam overflow rate. Data set numbers refer to Table 1.

theoretical estimation of D over what might be expected from independently measured values of μ_s.

For the first four sets of data, the temperature was 25°C. Conditions for the fifth set were apparently near ambient too. Accordingly, for all five sets, μ is taken as 8.94×10^{-3} (dyne)(sec.)/sq. cm., ρ as 1.00 g./cc., and g as 980 cm./sec.^{2}. There is evidence which indicates that μ_s, at least for Triton X-100, can vary markedly with temperature (*4, 5*). Impurities can also affect μ_s.

Figure 4 shows the test. The ordinate represents D/A calculated from theory, that is, from Equation (1), the wetness factor $(1 + 3D/G)$, and the solid curve of Figure 3. (Use of the dashed curve of Figure 3 would have yielded nearly the same results.) The abscissa of Figure 4 represents experimental values of D/A. The diagonal is the line of theoretical perfection.

Examination of Figure 4 reveals reasonably good agreement between the plotted points and the theoretical line. The average deviation of theory from experiment is listed in Table 1 for each set of data. The average deviation is 26% for the points involving the independently measured μ_s, and 23% for all the points. In view of the wide range of variables covered (more than 1,000-fold for D/A) and the experimental difficulties involved, this is deemed to be good agreement between theory and experiment.

Of course the agreement can be improved further by using foam determined μ_s for the first three sets of data as well. In this connection, the apparent effect of concentration in the interstitial liquid on μ_s, as exhibited in Figure 5 of reference 5, can be incorporated. However, this would reduce the independence of the test and so, for the sake of maximum objectivity, has not been done here.

COMPARISON FOR STEADY STATE DRAINAGE

As mentioned earlier, the theory also furnishes a theoretical steady state relationship between the bulk density of a stable stationary foam trapped in a column and the rate of a liquid stream fed on to the top of the foam so as to drain through to the bottom.

In this connection, two sets of experimental data are available to test the drainage theory. For the first set (*6, 8*) d was 0.37 cm. and A was 13.1 sq.cm., while for the second set (*4, 5*) d ranged from 0.10 to 0.19 cm. and A was 16.6 sq.cm. Aqueous Triton X-100 was employed for both sets. Accordingly, the independent μ_s value of 1.0×10^{-4} (dyne)(sec.)/cm. is used here for the test. The temperature was 25°C. Since the foam was stationary, G of course was zero.

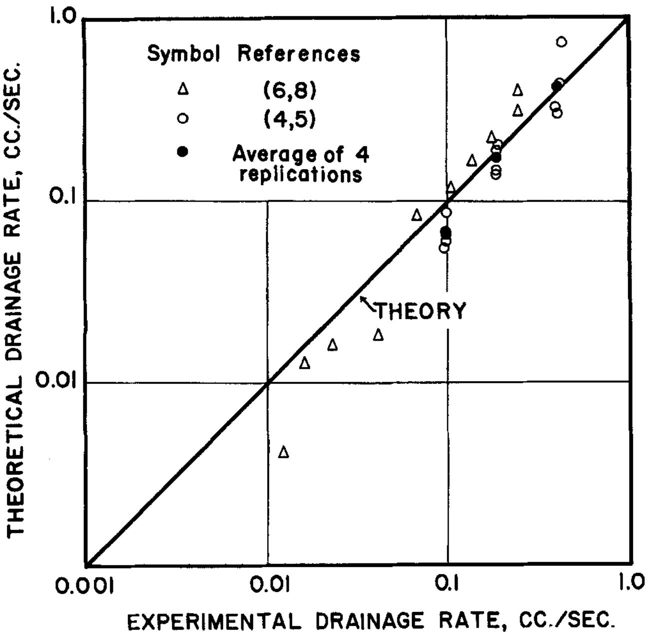

Fig. 5. Test of theory (7) for predicting the rate of steady state drainage through a stationary foam.

Figure 5 shows this drainage test. The ordinate represents the drainage rate predicted by theory from the measured bulk foam density and other pertinent information. This theoretical drainage rate was calculated in accordance with the information flow diagram of Figure 6 in which all notation, equation numbers, and figure numbers refer to reference 7. The aforementioned function ϕ does not enter the calculation in any way. Input information is encircled. Input t is of very small effect here. It was determined from Figure 5 of reference 8 but could just as well have been neglected.

The abscissa of the present Figure 5 represents the experimentally measured drainage rate. As before, the diagonal is the line of theoretical perfection. Inspection of this figure reveals fair agreement except at the lower drainage rates where experimental error is likely to be greatest. The average deviation of theory from experiment is 29%.

If one considers the experimental scatter apparent in the three sets of quadruplicated runs (shown as open circles in Figure 5), and then compares the three corresponding averages against the diagonal (so as to minimize the effect of individual error), better agreement between theory and experiment becomes immediately evident. The average deviation for these three averages is only 18%.

In view of the range of variables, the experimental difficulties, and the use of the independently determined value for μ_s, the test represented in Figure 5 appears to yield fair agreement between theory and experiment.

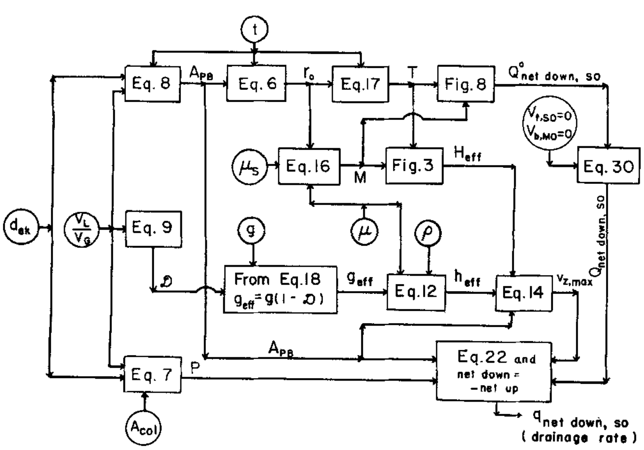

Fig. 6. Information flow diagram for predicting from theory (7) the rate of steady state drainage through a stationary foam. Input information is encircled. Since $v_{f,SO} = 0$ and $v_{b,MO} = 0$, $BO = MO = SO$. Notation, equations, and figures refer to reference 7.

CONCLUSIONS

The experimental results appear to support the theory. This is especially evident in light of the wide range of variables involved. Over more than a 400-fold range of foam overflow rate, the average deviation of theory from experiment is only 26%, even with μ_s independently determined.

ACKNOWLEDGMENT

This work was supported by U. S. Public Health Service Research Grant WP-00161.

NOTATION

A = horizontal cross-sectional area of a foam column, sq.cm.
D = volumetric flow rate of the entire foam overflow on a gas-free basis, cc./sec.
d = average bubble diameter, cm.
d_i = individual bubble diameter, cm.
G = volumetric flow rate of gas, cc./sec.
g = gravitational acceleration, cm./sec.^{2}
n_i = number of bubbles with diameter d_i
t = interstitial film (face) thickness, cm.
$v_{f,SO}$ = upward velocity of the aggregate of foam bubbles relative to a stationary observer, cm./sec.
$v_{z,\max,MO}$ = maximum velocity downward within a vertical capillary relative to the aggregate of foam bubbles, cm./sec.
$\bar{v}_{z,\max,MO}$ = average velocity downward within a vertical capillary relative to the aggregate of foam bubbles, cm./sec.

Greek Letters

μ = liquid viscosity, (dyne) (sec.)/sq.cm.
μ_s = surface viscosity, (dyne) (sec.)/cm.
ϕ = function
ρ = liquid density, g./cc.

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Manuscript received August 17, 1966; revision received November 21, 1966; paper accepted November 21, 1966.